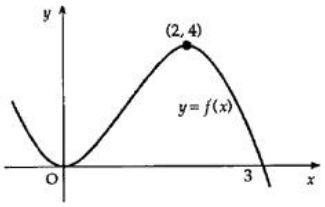
Higher Homework 9 – Calculus **Name** _____

1. The diagram shows the sketch of a cubic function f with stationary points at $(0,0)$ and $(2,4)$.

Sketch the graph of the derived function f'. **3**



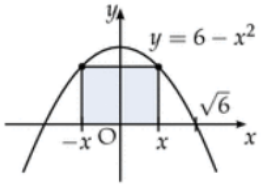
2. Find $\int \frac{x^2 - 5}{x\sqrt{x}}\, dx$ **4**

3. If $f(x) = \cos^2 x - \frac{2}{3x^2}$, find $f'(x)$. **4**

4. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.

If $f'(x) = \sin 3x$, express y in terms of x. **4**

5. A rectangle is formed under the graph of $y = 6 - x^2$, as shown in the diagram.

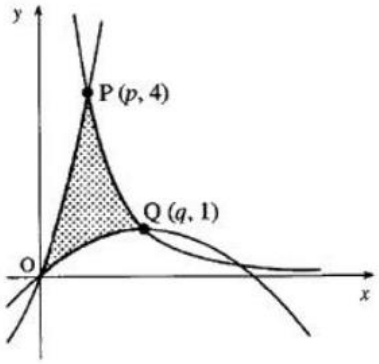


(a) Show that the area of the rectangle can be given by:

$$A(x) = 12x - 2x^3 \text{ for } 0 < x < \sqrt{6}$$ **2**

(b) Hence find the value of x which maximises the area of the rectangle, and the corresponding area. **5**

6. The origin O and the points P and Q are the vertices of a curved "triangle" which is shaded in the diagram.



The sides lie on the curves with equations:

$$y = x(x+3), \quad y = x - \frac{1}{4}x^2 \text{ and } y = \frac{4}{x^2}.$$

(a) P and Q have coordinates $(p, 4)$ and $(q, 1)$.
Find the values of p and q. **2**

(b) Calculate the shaded area. **7**