Higher Homework 5 **Name** ______________________________

1. A sequence is defined by the recurrence relation $u_{n+1} = au_n + 22$, $u_0 = 1$, with $-1 < a < 1$.

(a) Find the value of a for which the sequence has a limit of 33. **3**

(b) Show that the sequence cannot have a limit of 10. **3**

2. At 12 noon a hospital patient is given a pill containing 50 units of antibiotic.
By 1 pm the number of units in the patient's body has dropped by 12%.
By 2 pm a further 12% of the units remaining in the body at 1 pm is lost.

(a) If this fall-off rate is maintained, find the number of units of antibiotic remaining at 6 pm. **4**

A doctor considers prescribing a course of treatment which involves a patient taking one of these pills every 6 hours over a long period of time. The doctor knows that more than 100 units of antibiotic in the body are dangerous.

(b) Is it safe for the doctor to prescribe the treatment? Give reasons for your answer. **6**

3. Two sequences are defined by the recurrence relations

$$u_{n+1} = 0.2u_n + p, \quad u_0 = 1 \qquad \text{and}$$

$$v_{n+1} = 0.6v_n + q, \quad v_0 = 1$$

If both sequences have the same limit, express p in terms of q. **3**

4. A sequence is defined by the recurrence relation $u_{n+1} = ku_n + k$, where $u_0 = 1$ and k is a constant with $k \neq 0$.

(a) Find expressions for u_1 and u_2. **2**

(b) Show that if $u_2 = 0$, the sequence has a limit. **3**

5. A sequence is defined by the recurrence relation $u_{n+1} = au_n + b$ with $u_0 = 1$, where a and b are constants. Given that $u_1 = 19$ and $u_2 = 13$,

(a) Find the values of a and b . **3**

(b) Explain why the sequence has a limit and find this limit. **3**

Answers to Higher Homework 5 – Sequences

1(a) $a = \frac{1}{3}$

1(b) For a limit of 10, $a = -1.2$. Since $-1.2 < 1$, no limit exists.

2(a) 23.22 units remain

2(b) In the long term, the units will approach a limit of 93.35 and since $93.35 < 100$, the treatment is safe.

3. $p = 2q$

4(a) $u_1 = 2k$ and $u_2 = 2k^2 + k$

4(b) If $u_2 = 0$ then $k = -\frac{1}{2}$ and since $-1 < -\frac{1}{2} < 1$ then the sequence has a limit

5(a) $a = -\frac{1}{3}$ and $b = 19\frac{1}{3}$

5(b) Limit exists as $-1 < -\frac{1}{3} < 1$ with limit = $14\frac{1}{2}$