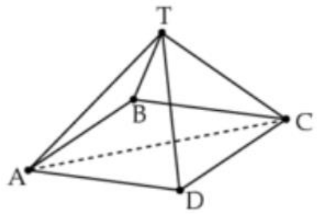
Higher Maths - Homework 3 **Name** __________________________________

1. Find the acute angle between the vectors $\underset{\sim}{\mathbf{u}} = \begin{pmatrix} 6 \\ \sqrt{3} \\ 3 \end{pmatrix}$ and $\underset{\sim}{\mathbf{v}} = \begin{pmatrix} -\frac{1}{2} \\ 4 \\ -1 \end{pmatrix}$. **4**

2. The square based pyramid ABCDT is shown. All of the edges of ABCDT have length 4 units.



(a) Find the exact value of $\cos \mathrm{TAC}$. **2**

(b) Hence find the exact value of $\overrightarrow{AT}.\left(\overrightarrow{AB}+\overrightarrow{AC}\right)$. **4**

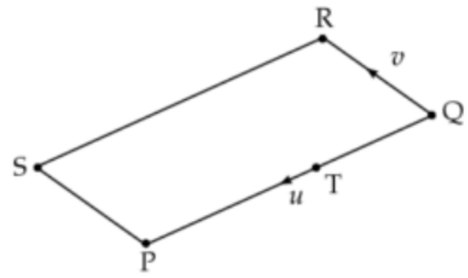
3. The vectors $\underset{\sim}{\mathbf{a}}$, $\underset{\sim}{\mathbf{b}}$ and $\underset{\sim}{\mathbf{c}}$ are defined as follows:

$\underset{\sim}{\mathbf{a}} = 2\underset{\sim}{\mathbf{i}} - \underset{\sim}{\mathbf{k}}$, $\qquad \underset{\sim}{\mathbf{b}} = \underset{\sim}{\mathbf{i}} + 2\underset{\sim}{\mathbf{j}} + \underset{\sim}{\mathbf{k}}$, $\qquad \underset{\sim}{\mathbf{c}} = -\underset{\sim}{\mathbf{j}} + \underset{\sim}{\mathbf{k}}$

(a) Evaluate $\underset{\sim}{\mathbf{a}}.\underset{\sim}{\mathbf{b}}$ and $\underset{\sim}{\mathbf{a}}.\underset{\sim}{\mathbf{c}}$. **3**

(b) From your answer to part (a), what can you say about the vector $\underset{\sim}{\mathbf{b}} + \underset{\sim}{\mathbf{c}}$? **2**

4. The parallelogram PQRS is shown in the diagram.



The vectors $\underset{\sim}{\mathbf{u}}$ and $\underset{\sim}{\mathbf{v}}$ represent line segments $\overrightarrow{QP}$ and $\overrightarrow{QR}$ respectively, and are such that $|\underset{\sim}{\mathbf{u}}| = 5$, $|\underset{\sim}{\mathbf{v}}| = 2$ and $\underset{\sim}{\mathbf{u}}.\underset{\sim}{\mathbf{v}} = 5$. The point T divides $\overrightarrow{QP}$ in the ratio 2 : 3.

(a) Express $\overrightarrow{RP}$ and $\overrightarrow{RT}$ in terms of $\underset{\sim}{\mathbf{u}}$ and $\underset{\sim}{\mathbf{v}}$. **3**

(b) Hence evaluate $\overrightarrow{RP}.\overrightarrow{RT}$. **3**

Answers to Higher Homework 3 – Vectors

1. $\theta = 88.2^\circ$

2(a) $\cos TAC = \dfrac{\sqrt{2}}{2}$

2(b) $8 + 8\sqrt{2}$

3(a) $\underset{\sim}{\mathbf{a}}.\underset{\sim}{\mathbf{b}} = 1$ and $\underset{\sim}{\mathbf{a}}.\underset{\sim}{\mathbf{c}} = -1$

3(b) Since $\underset{\sim}{\mathbf{a}}.\left(\underset{\sim}{\mathbf{b}} + \underset{\sim}{\mathbf{c}}\right) = 0$ then vector $\underset{\sim}{\mathbf{b}} + \underset{\sim}{\mathbf{c}}$ is perpendicular to vector $\underset{\sim}{\mathbf{a}}$

4(a) $\overrightarrow{\mathrm{RP}} = \underset{\sim}{\mathbf{u}} - \underset{\sim}{\mathbf{v}}$ and $\overrightarrow{\mathrm{RT}} = \dfrac{2}{5}\underset{\sim}{\mathbf{u}} - \underset{\sim}{\mathbf{v}}$

4(b) $\overrightarrow{\mathrm{RP}}.\overrightarrow{\mathrm{RT}} = 7$